Section 1.4

Definition of Continuity

Continuity at a Point: A function f is **continuous at c** when these three conditions are met.

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an open interval: A function is **continuous on an open interval (a, b)** when the function is continuous at each point in the interval. A function that is continuous on the entire real number line $(-\infty, \infty)$ is **everywhere continuous**.

Discontinuity: If a function f is defined on an open interval I, and f is not continuous at some value $x = c$ on I, then f has a discontinuity at c. A discontinuity at c is called **removable** when f can be made continuous by appropriately (re)defining $f(c)$ (otherwise, it is **nonremovable**).

1) Discuss the continuity of each function, that is, state any values of x for which the function is not continuous, and state what type of discontinuity it is (removable or nonremovable).

   a) $f(x) = \frac{x}{x^2 - x - 6}$

   b) $g(x) = \frac{x+2}{x^2 - 4}$

   c) $h(x) = \begin{cases} 1 - x, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

   d) $y = \tan x$

The Existence of a Limit: Let f be a function, and let c and L be real numbers. The limit of $f(x)$ as x approaches c is L if and only if

$$\lim_{x \to c^-} f(x) = L \quad \text{and} \quad \lim_{x \to c^+} f(x) = L$$

2) Find the following limits:

a) $\lim_{x \to -3^+} \sqrt{x^2 - 9}$

b) $\lim_{x \to 2^-} \left[\!\left[\frac{1}{2} x \right]\!\right]$

3) Discuss the continuity of $f(x) = \sqrt{x^2 - 4}$.

4) Determine the interval(s) on which the following functions are continuous.

a) $f(x) = \csc x$

b) $f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Intermediate Value Theorem: If f is continuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, there is at least one number c in $[a, b]$ such that $f(c) = k$.

5) Use the Intermediate Value Theorem to show that the function $f(x) = x^3 - 2x^2 + 2$ has a zero on the interval $[-1, 0]$.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #4, 9, 13, 19, 28, 47, 53, 56, 61, 66, 95